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CONDITIONALLY PERIODIC MOTIONS IN THE ATTRACTION FIELD OF A ROTATING TRIAXIAL ELLIPSOID*

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Conditionally periodic solutions are constructed in the neighbourhood of previously derived steady-state solutions of the problem of the motion of a material point in the attraction field of a rotating triaxial ellipsoid, when the average motion of the material point and the ellipsoid angular velocity of rotation are commensurable.

The problem of the motion of a material point in the attraction field of a homogeneous triaxial ellipsoid (or non-homogeneous ellipsoid with ellipsoidal layers of the same density) uniformly rotating about one of its principle central axes of inertia was previously considered in /1–4/. Families of steady-state solutions (motions) were obtained in /4/, when the average motion of the material point and the angular velocity of the ellipsoid are commensurable, and the existence of conditionally periodic solutions (motions) in their neighbourhood was proved.

The purpose of the present paper is to construct conditionally periodic solutions of this problem, and to establish the properties of such motions of a material point. The method of constructing the conditionally periodic solutions of canonical systems of differential equations is used when the frequencies are strictly commensurable /5/.

1. The equations of motion and the Hamiltonian. Consider a rotating system of coordinates $OXYZ$, whose axes lie along the principal central axes of inertia of a triaxial ellipsoid. The OX and OY axes coincide with the minor and major axes of the ellipsoid equatiorial cross section, respectively, and the OZ axis coincides with the ellipsoid axis or rotation.

The material point equations of motion in the triaxial ellipsoid attraction field can be represented in the rotating system of coordinates $OXYZ$ in the form

$$dx_j/d\tau = \partial F'/\partial y_j,\ dy_j/d\tau = -\partial F'/\partial x_j,\ j = 1,\ 2,\ 3 \tag{1.1}$$

$$x_1 = L = \sqrt{a},\ y_1 = l = M \tag{1.2}$$

$$x_2 = G = \sqrt{a(1-e^2)},\ y_2 = g = \omega$$

$$x_3 = H = \sqrt{a(1-e^2)}\cos i,\ y_3 = h = \Omega - \omega_0\tau$$

where x_j, y_j are the Delaunay canonical variables, a, e, i, M, Ω and ω represent the conventional system of Keplerian osculating elements, ω_0 is the angular velocity of rotation of the ellipsoid, and τ is the dimensionless time, while the Hamiltonian F' has the following structure:

$$F' = F_0' + \varkappa F_1',\ \varkappa \ll 1 \tag{1.3}$$

$$F_0 = \frac{1}{2}x_1^{-2} + x_3\omega_0,\quad F_1' = \sum_{\|\mathbf{k}\|\geqslant 0} A_{\mathbf{k}}(\mathbf{x})\cos(\mathbf{k},\mathbf{y})$$

$$\mathbf{k} = (k_1,\ k_2,\ k_3),\ (\mathbf{k},\ \mathbf{y}) = k_1y_1 + k_2y_2 + k_3y_3,\ \|\mathbf{k}\| = |k_1| + |k_2| + |k_3|$$

The explicit form of the coefficients $A_{\mathbf{k}}(\mathbf{x})$ will be given later.

2. Steady-state motions. To investigate the steady-state conditionally periodic motions of a material point in the triaxial ellipsoid attraction field, defined by a commensurability of the form $n/\omega_0 = s/p$ (n is the material point mean motion, ω_0 is the angular velocity of rotation of the ellipsoid, and $s = p + q,\ p,\ q$ are integers), it is more convenient to use the system of variables /4/

$$X_1 = sL - pH,\quad Y_1 = l/s \tag{2.1}$$

$$X_2 = G - H,\quad Y_2 = g$$

$$X_3 = H,\quad Y_3 = pl/s + g + h$$

the substitution of which into (1.1) yields

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$$dX_j / d\tau = \partial F / \partial Y_j,\ dY_j / d\tau = -\partial F / \partial X_j,\ j = 1, 2, 3 \tag{2.2}$$

$$F = F_0 + \varkappa F_1 \tag{2.3}$$

$$F_0 = \frac{1}{2} s^2 (X_1 + pX_3)^{-2} + \omega_0 X_3$$

$$F_1 = \sum_{\|\mathbf{k}\| \geqslant 0;\ m=0,\ \pm 1,\ \pm 2} A_{\mathbf{k}}^* (X) \cos[(k_1 s + pm) Y_1 + k_2 Y_2 + k_3 Y_3]$$

Using the method described in /5/ we separate from the perturbing part F_1 of Hamiltonian F the secular part F_1^{sec}, the resonant part (including the long-period part) F_1^{res}, and the short-period part F_1^{sp}:

$$F_1^{sec}(X_j) = \frac{s^6}{12} (X_1 + pX_3)^{-6} (3\alpha - 1) C_0^{-3,0} (X_j) \tag{2.4}$$

$$F_1^{res} (X_j, Y_2, Y_3) = \frac{3}{4} \varkappa b_{22} s^6 (X_1 + pX_3)^{-6} \sum_{\nu=1}^{3} \Phi_\nu$$

$$\Phi_1 = (1 - \alpha^2) C_{k_1}^{-3,0} \cos[(k_1 s - 2p) Y_1 - 2Y_2 + 2Y_3]$$

$$\Phi_2 = \frac{1}{2} (1 + \alpha)^2 (C_{k_1}^{-3,2} + S_{k_1}^{-3,2}) \cos[(k_1 s - 2p) Y_1 + 2Y_3]$$

$$\Phi_3 = \frac{1}{2} (1 - \alpha)^2 (C_{k_1}^{-3,2} - S_{k_1}^{-3,2}) \cos[(k_1 s - 2p) Y_1 - 4Y_2 + 2Y_3]$$

$$\alpha = X_3/(X_2 + X_3)$$

$$F_1^{sp} (X_j, Y_j) = \sum_{\|\mathbf{k}\|>0}{}' A_{\mathbf{k}}^* (X_j) \cos[(k_1 s + mp) Y_1 + k_2 Y_2 + k_3 Y_3] \tag{2.5}$$

where the primes in Eq.(2.5) indicate that the vector $\mathbf{k}$ takes only non-resonant values, in particular $(k_1 s + mp) \neq 0$.

To determine the steady solutions of (2.2) with the Hamiltonian (2.3)–(2.5) and to construct the conditionally periodic solutions in their neighbourhood we introduce, using a transformation similar to a Zeipel transformation, the canonical replacement of variables by the formulas

$$P = X + \varkappa \frac{\partial S}{\partial Y},\ Y = Q + \varkappa \frac{\partial S}{\partial P},\ P = (P_1, P_2, P_3),\ Q = (Q_1, Q_2, Q_3) \tag{2.6}$$

where P and Q are the new canonical system of variables, and $\varkappa S\ (P, Y)$ is a small generating function. The function that generates transformation $S\ (P,\ Y)$ is selected so that the new Hamiltonian does not contain short-periodic terms $O\ (\varkappa)$ (for details see /5/).

Substituting (2.6) into (2.2), we obtain

$$\frac{dP}{d\tau} = \frac{\partial F^*}{\partial Q},\quad \frac{dQ}{d\tau} = -\frac{\partial F^*}{\partial P} \tag{2.7}$$

$$F^* (P,\ Q) = F_0^* (P) + \varkappa F_1^* (P,\ Q_2,\ Q_3) + \varkappa^2 F_2^* (P,\ Q)$$

$$F_1^* (P, Q_2, Q_3) = F_1^{sec} (P) + F_1^{res} (P, Q_2, Q_3)$$

To obtain the explicit form of the functions F^* and F_1^*, and others it is sufficient to replace X by P and Y by Q in (2.3) and (2.4).

Neglecting in Eqs.(2.7) the short-period terms $O\ (\varkappa^2)$ (the secular and resonance components $\varkappa^2 F_2^*\ (P, Q)$ can be included in the functions F_1^{sec} and F_1^{res}, respectively), we will seek steady-state solutions of (2.7) in the form

$$\bar{P}_j = P_{j0} = \text{const},\quad j = 1, 2, 3 \tag{2.8}$$

$$\bar{Q}_i = Q_{i0} = \text{const},\quad i = 2, 3;\ \bar{Q}_1 = Q_{10} + \omega_1 \tau$$

The necessary conditions for steady-state solutions of the form (2.8) to exist are as follows /6/:

$$\frac{d\bar{P}_j}{d\tau} = \frac{\varkappa \partial F_1^*}{\partial Q_j} = 0,\quad j = 1, 2, 3 \tag{2.9}$$

$$\frac{d\bar{Q}_1}{d\tau} = -\frac{\partial \bar{F}}{\partial P_1} = \omega_1,\quad \frac{d\bar{Q}_2}{d\tau} = -\frac{\nu \partial F_1^*}{\partial P_2} = 0$$

$$\frac{d\bar{Q}_3}{d\tau} = -\frac{\partial \bar{F}}{\partial P_3} = 0\quad (\bar{F} = F_0^* + \varkappa F_1^*)$$

Let us consider conditions (2.9) in succession.

The first condition is automatically satisfied when $j = 1$ (since the function $\bar{F}$ is independent of the rapid variable Q_1), and when $j = 2, 3$ it is satisfied, if $\bar{Q}_2 = Q_{20} = k_1^* \pi/2$

and $\bar{Q}_3 = Q_{30} = k_2^* \pi/2$, and k_1^*, k_2^* are integers. The fourth condition is satisfied, if the satellite mean motion is selected from the relation

$$n = sp^{-1} [\omega_0 + \varkappa (ps^{-1}\partial F_1^*/\partial L + \partial F_1^*/\partial H)] \tag{2.10}$$

A computer is used to check the third condition, which we will write in the form $\partial F_1^*/\partial P_2 = \Phi (e, i) = 0$, and the solution defines the dependence between P_2 and P_3 (actually between e and i) in the steady-state solution (see /4/). Finally, the second condition yields the expression

$$\omega_1 = p^{-1} (\omega_0 + \varkappa\partial F_1^*/\partial H) \tag{2.11}$$

The necessary conditions (2.9) can, thus, all be satisfied (and this indicates their sufficiency), and steady-state solutions of the form (2.8) do exist.

Steady-state motions of a material point that correspond to the steady-state solution (2.8) have, in Keplerian elements, the following properties (generally $e \neq 0, i \neq 0$):

1) the material point moves over an elliptic orbit of constant dimensions and inclination to the ellipsoid equatorial plane with an average motion n somewhat different from the exactly commensurable form $s/p\omega_0$, and the major semi-axis of the orbit a depends on the form of the commensurability s/p, and e and i are related by the equation $\Phi (e, i) = 0$ and depend on the form of commensurability;

2) the line of apsides of the orbit is stationary in the rotating system of coordinates $OXYZ$ and is constantly oriented along the meridian of the minor or major semiaxes, respectively, of the ellipsoid equatorial cross section;

3) the orbit line of nodes processes in the rotating system of coordinates at a low velocity $\Omega^{\cdot} = - \varkappa\partial F_1^*/\partial H$;

4) the mean longitude Q_3 of the material point remains constant in the rotating system of coordinates coinciding with the meridian of the major or minor semiaxes of the ellipsoid equatorial cross section;

5) the rapid angular variable Q_1 varies in such a way that the material point, when the period $T_1 = 2\pi/\omega_1$ expires, appears on the same meridian to which the value of Q_{10} corresponds; at the same time the period of rotation of the ellipsoid ($T_0 = 2\pi/\omega_0$), the period of rotation of the material point ($T = 2\pi/n$) , and the period of variation of the rapid variable ($T_1 = 2\pi/\omega_1$) are connected by the following relations:

$$T_1 = pT_0 (1 + O (\varkappa)), \quad T_1 = sT (1 + O (\varkappa)), \quad (O (\varkappa) = - \varkappa\omega_0^{-1} \partial F_1^*/\partial H)$$

As a result of the motion described, the material point is on the same original meridian after p rotations of the ellipsoid about its own axis or after s rotations of the material point on the orbit.

We will now construct conditionally periodic solutions (motions) in the neighbourhood of the steady-state solutions (motions) obtained.

3. The generating function. The short-period terms F_1^{sp} of the Hamiltonian were eliminated using the generating function $S (P, Y)$ which is dependent on the structure and form of F_1^{sp}.

To obtain conditionally-periodic solutions in several systems of elements (variables) we will write $\varkappa F_1^{sp}$ using the Kepler position elements a, e, and i, the Delaunay angular variables l , g and h, and also the variables (X, Y).

As the result we obtain

$$\begin{aligned} &\varkappa F_1^{sp} (a, e, i, l, g, h) = \varkappa \sum_{\|\mathbf{k}_1\|>0} A_{\mathbf{k}_1} (a, e, i) \cos (kl + 2\nu g) + \\ &\quad \varkappa^2 \sum_{\|\mathbf{k}_2\|>0} B_{\mathbf{k}_2} (a, e, i) \cos (kl + 2\nu g + 2\varepsilon h) \\ &\mathbf{k}_1 = (k,\ 2\nu),\ \mathbf{k}_2 = (k,\ 2\nu,\ 2\varepsilon),\ \nu = 0; \pm 1,\ \varepsilon = \pm 1 \\ &-\infty < k < \infty \\ &A_{k,0} = \frac{3 \cos^2 i - 1}{12a^3} C_k^{-3,0} (e) \\ &A_{k,\pm 2} = \frac{\sin^2 i}{8a^3} [C_k^{-3,2} (e) \pm S_k^{-3,2}(e)] \\ &B_{k,0,\pm 2} = \frac{3b_{22} \sin^2 i}{4a^3} C_k^{-3,0} (e) \\ &B_{k,\pm 2,\pm 2} = \frac{3b_{22} (1 \pm \cos i)^2}{8a^3} [C_k^{-3,2} (e) \pm S_k^{-3,2} (e)] \end{aligned} \tag{3.1}$$

where $C_k^{n,m} (e)$, $S_k^{n,m} (e)$ are known expansions in powers of e /7/.

To obtain now the explicit form of the function $\varkappa F_1^{sp}$ it is sufficient to substitute X, Y, for $a, \ldots, h$ in (3.1) using (1.2) and (2.1).

Using (3.1) for the short-period part of $\varkappa F_1^{sp}$ we select the generating function S of transformation (2.6) in the form of the sum

$$S = S_1 + S_2 \tag{3.2}$$

where

$$S_1(a,e,i,l,g,h) = \varkappa \sum_{\|\mathbf{k}_1\|=1}^{\infty}{}'' S_{\mathbf{k}_1}(a,e,i)\sin(kl + 2\nu g) \tag{3.3}$$

$$S_2(a,e,i,l,g,h) = \varkappa^2 \sum_{\|\mathbf{k}_2\|=1}^{\infty}{}'' S_{\mathbf{k}_2}(a,e,i)\sin(kl + 2\nu g + 2\varepsilon h)$$

or

$$S_1(P,Y) = \varkappa \sum_{\|\mathbf{k}_1\|=1}^{\infty}{}' S_{\mathbf{k}_1}(P)\sin(ksY_1 + 2\nu Y_2) \tag{3.4}$$

$$S_2(P,Y) = \varkappa^2 \sum_{\|\mathbf{k}_2\|=1}{}' S_{\mathbf{k}_2}(P)\sin[(ks - 2p\varepsilon)Y_1 + \nu Y_2 + \varepsilon(Y_3 - Y_2)]$$

$$\nu = 0;\ \pm 1,\quad \varepsilon = \pm 1$$

The primes on the summation sign indicate that they contain no long-period and resonance terms (i.e. $k \neq 0$ in (3.3) and $ks - 2p\varepsilon \neq 0$ in the first of expressions (3.4)), that are already included in the function $\varkappa F_1^{res}$.

Taking the small parameter as $\varkappa \sim 10^{-3}$, we will confine ourselves to constructing conditionally periodic solutions to a first approximation (in the steady-state solutions the secular, long-period, and resonance terms of the perturbing function $O(\varkappa^2)$ are taken into account). In this connection we shall only consider the part $S_1 = O(\varkappa)$ in the generating function S and retain a finite number of harmonics $k \leqslant N = 10$ (as k increases the amplitude of the corresponding harmonic decreases).

Taking the above into account, we have

$$\varkappa S = \varkappa S_1 = \varkappa \sum_{k=1}^{10}{}' S_k(a,e,i)\sin(kl + 2\nu g) = \tag{3.5}$$

$$\varkappa \sum_{k=1}^{10}{}' S_k(P)\sin(ksY_1 + 2\nu Y_2);\quad \nu = 0, \pm 1$$

$$S_k(a,e,i) = -A_{\mathbf{k}_1}(a,e,i) \mid (\mathbf{k}_1, \partial F \mid \partial\alpha);\quad \alpha = a, e, i$$

$$S_k(P) = -A_{\mathbf{k}_1}(P) \mid (\mathbf{k}_1, \partial F^* \mid \partial P)$$

$$F^*(P) = F_0{}^*(P) + \varkappa F_1^{sec}(P)$$

Taking the last relation into consideration, we can write the coefficients in their final form

$$S_k(P) = -A_{\mathbf{k}_1}(P) / nk + O(\varkappa) \tag{3.6}$$

4. The conditionally-periodic solutions. The steady-state solutions obtained and described in Sect.2 and, also, the explicit form of the generating function of transformation (2.6), enable us to write conditionally periodic solutions in several systems of variables.

The conditionally periodic solutions in (P,Q) variables. Using the general formulas of conditionally periodic solutions /5, 6/, we obtain $\bar{P}, \bar{Q}$ the values of the variables P and Q in the steady-state solution (see (2.8))

$$P_i(\tau) = \bar{P}_i + \varkappa \sum_{k=1}^{10}{}'' f_{P\mathbf{k}}(\bar{P}_i)\cos(\mathbf{k}, \bar{Q}_j);\quad i = 1,2,3;\quad j = 1,2 \tag{4.1}$$

$$Q_i(\tau) = \bar{Q}_i + \varkappa \sum_{k=1}^{10}{}'' f_{Q\mathbf{k}}(\bar{P}_i)\sin(\mathbf{k}, \bar{Q}_j)$$

$$\bar{P}_i = P_{i0},\quad \bar{Q}_1 = Q_{10} + \omega_1\tau,\quad \bar{Q}_2 = Q_{20},\quad \bar{Q}_3 = Q_{30}$$

$$(\mathbf{k}, \bar{Q}_j) = [ks(Q_{10} + \omega_1\tau) + 2\nu Q_{20}]$$

$$\omega_1 = \frac{1}{p}\left(\omega_0 + \varkappa\frac{\partial F_1{}^*(P_i, Q_2, Q_3)}{\partial P_3}\right)\Big|_{P_i = \bar{P}_i,\ Q_i = \bar{Q}_i}$$

$$f_{P\mathbf{k}}(\bar{P}_i) = kS_k(P)\big|_{P_i = \bar{P}_i},\quad f_{Q\mathbf{k}}(\bar{P}_i) = -\frac{\partial S_k(P)}{\partial P}\Big|_{P_i = \bar{P}_i}$$

The conditionally periodic solutions in mixed variables. Using the expressions

$$n = s^3 (X_1 + pX_3)^{-3},\ e = [1 - s^2 (X_2 + X_3)^2 (X_1 + pX_3)^{-2}]^{1/2}$$

$$i = \arccos\left(\frac{X_3}{X_2 + X_3}\right);\quad Y_1 = \frac{l}{s},\quad Y_2 = g,\quad Y_3 = \frac{pl}{s} + g + h$$

derived from (1.2) and (2.1), we can write the conditionally periodic solutions in mixed variables

$$n = \bar n - 3\varkappa \bar n^{4/3} s^{-1} \frac{\partial S}{\partial Y_1},\quad e = \bar e + \tag{4.2}$$

$$\varkappa \bar e^{-1} \bar n^{1/3} \sqrt{1 - \bar e^2}\left(s^{-1}\sqrt{1 - \bar e^2}\frac{\partial S}{\partial Y_1} - \frac{\partial S}{\partial Y_2}\right)$$

$$i = \bar i + \varkappa (1 - \bar e^2)^{-1/2} \bar n^{1/3} \operatorname{ctg} \bar i \frac{\partial S}{\partial Y_2}$$

$$Y_1 = \bar Q_1 + 3\varkappa\left[s^{-1}\bar n^{-4/3}\frac{\partial S}{\partial n} - \frac{\bar n^{1/3}(1 - \bar e^2)}{3s\bar e}\frac{\partial S}{\partial e}\right]$$

$$Y_2 = \bar Q_2 + 3\varkappa\left[\frac{p}{s\bar n^{4/3}}\frac{\partial S}{\partial n} - \frac{\bar n^{1/3}(1 - \bar e^2)^{1/2}[p(1 - \bar e^2) - s]}{3s\bar e}\frac{\partial S}{\partial e} - \right.$$

$$\left.\frac{\bar n^{1/3}(\cos\bar i - 1)}{3\sin\bar i\,(1 - \bar e^2)^{1/2}}\frac{\partial S}{\partial i}\right]$$

$$Y_3 = \bar Q_3 + \varkappa\left[\frac{1}{\bar e\sqrt{1 - \bar e^2}}\frac{\partial S}{\partial e} - \frac{\bar n^{1/3}\operatorname{ctg}\bar i}{\sqrt{1 - \bar e^2}}\frac{\partial S}{\partial i}\right]$$

where $\bar\alpha_j$ is the value of the element α_j in the steady-state (2.8) $(\alpha_j = n, e, i, Q_j)$.

The conditionally periodic solution in Kepler-Delaunay variables.

$$n = \bar n - 3\varkappa \bar n^{4/3}\frac{\partial S}{\partial l},\quad e = \bar e + \varkappa \bar e^{-1} \bar n^{1/3}(1 - \bar e^2)^{1/2}\cdot \tag{4.3}$$

$$\left[(1 - \bar e^2)^{1/2}\frac{\partial S}{\partial l} - \frac{\partial S}{\partial g}\right]$$

$$i = \bar i + \varkappa \bar n^{1/3}(1 - \bar e^2)^{-1/2}\operatorname{ctg}\bar i\frac{\partial S}{\partial g}$$

$$l = \bar l + 3\varkappa\left[\bar n^{4/3}\frac{\partial S}{\partial n} - \frac{\bar n^{1/3}(1 - \bar e^2)}{3\bar e}\frac{\partial S}{\partial e}\right]$$

$$g = \bar g + \varkappa \bar n^{1/3}\left[\frac{\sqrt{1 - \bar e^2}}{\bar e}\frac{\partial S}{\partial e} - \frac{\operatorname{ctg}\bar i}{\sqrt{1 - \bar e^2}}\frac{\partial S}{\partial i}\right]$$

$$h = \bar h + \varkappa \bar n^{1/3}\sin^{-1}\bar i\,(1 - \bar e^2)^{-1/2}\frac{\partial S}{\partial i}$$

$$\bar l = l_0 + \omega_1\tau,\quad \bar g = g_0,\quad \bar h = h_0 + \omega_3\tau$$

where ω_1 is defined by (2.10), and $\omega_3 = -p\omega_1/s$. In addition we must substitute into the derivatives $\partial S/\partial\alpha_j$ the steady-state values $\bar\alpha_j = \bar n, \bar e, \bar i, \bar l, \bar g$ and $\bar h$.

The conditionally periodic motions of a material point defined by solutions (4.1), (4.2), and (4.3) have virtually the same character in all systems of variables: short-periodic oscillations of amplitude $O(\varkappa)$ are imposed on the steady motions. The difference is solely that in certain systems of variables the separate angular variables have constant steady motions (for instance, the variables Q_2 and Q_3 in (4.1) and (4.2) and the variable g in (4.3)) while all the remaining angular variables are linear functions of time.

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